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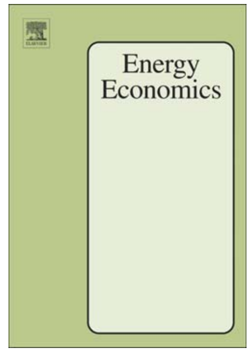

Resource extraction with a carbon tax and regime switching prices: Exercising your options

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Resource extraction with a carbon tax and regime switching prices: Exercising your options

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Abstract

This paper develops a model of a profit maximizing firm with the option to exploit a non-renewable resource, choosing the timing and pace of development. The resource price is modelled as a regime switching process, which is calibrated to oil futures prices. A Hamilton-Jacobi-Bellman equation is specified that describes the profit maximization decision of the firm. The model is applied to a problem of optimal investment in a typical oils sands *in situ* operation, and solved for critical levels of oil prices that would motivate a firm to make the large scale investment needed for oil sands extraction, as well to operate, mothball or abandon the facility. Regime shifts can have an important effect on the optimal timing of investment and extraction. The paper examines the effect of several carbon tax schemes on optimal timing of construction, production and abandonment. A form of Green Paradox is identified.

Keywords: non-renewable natural resources, oil sands, optimal control, HJB equation, carbon tax, regime switching, *JEL codes:* Q30, Q40, C61, C63

1. Introduction

Commodity prices are typically highly volatile and characterized by cycles of boom and bust. Not surprisingly, investments in resource extraction tend to mirror these cycles. One example can be found in investment in the high cost oil sands reserves in Alberta, Canada. Beginning in the 1970's, investment in the extraction of the oil sands was an on-again

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off-again proposition depending on the strength of oil prices. World crude prices since 1986 and capital expenditure in the oil sands since 1973 are shown in Figures 1 and 2 respectively. Buoyant oil prices in the past decade up until mid-2014 have been associated with unprecedented investment in oil sands extraction. The collapse of oil prices in the latter part of 2014 resulted in many cancellations and delays of spending plans and total capital expenditures dropped sharply in 2015. However, oil sands production (Figure 3) has shown a fairly steady upward trend with no indication that producing projects have curtailed production in response to low oil prices.

The run-up in oil prices and resultant strong investment in oil sands extraction of the past two decades raised concerns nationally and internationally about the impact of such large scale operations on the local environment including water quality for nearby residents, wildlife habitat and general ecosystem health. Added to the more localized environmental impacts are widespread concerns about increased carbon emissions from expanding production from oil sands reserves, which has a high carbon content compared to other sources of crude production (Lattanzio, 2014). The Alberta government was criticized for not having adequate regulatory oversight in place to ensure that environmental impacts are kept at acceptable levels. Oil sands operators have felt the pressure of strong negative public opinion expressed around the world and there is a sentiment that they have lost their "social license to operate".[2]

In the past decade, development of oil sands reserves has been threatened by new sources of supply such as oil and natural gas from shale deposits, which have been made accessible by newly developed technologies. The dramatic fall in the price of oil in late 2014 is a reflection

[2]The environmental concerns raised by oil sands extraction are well documented by the Pembina Institute. *http://www.pembina.org/*. The alarm over a lack of regulatory oversight was raised in the 2006 report "Canada's Oil Sands Rush" (Woynillowicz et al., 2005) and the 2008 report "Taking the Wheel" (Severson-Baker, 2008).

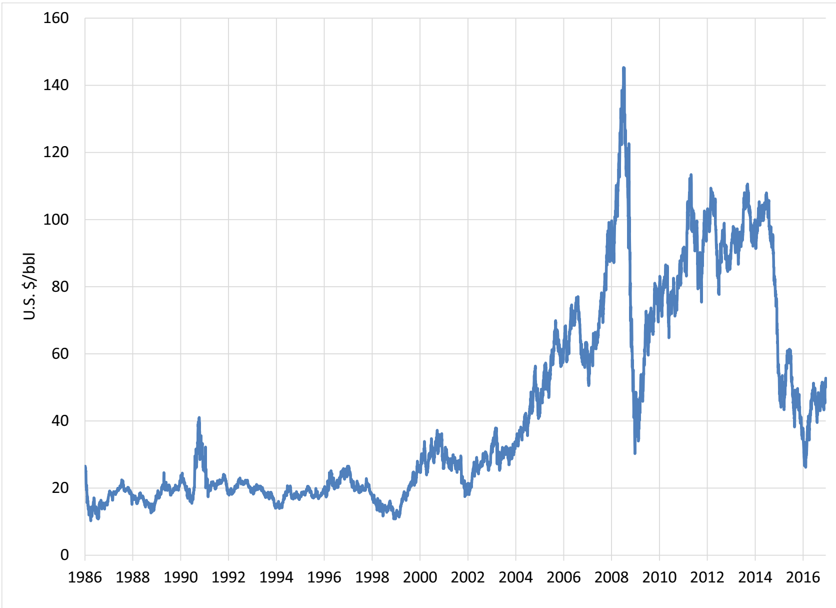


Figure 1: West Texas Intermediate Crude Oil Spot Price, U.S. $/barrel, Daily, Jan 1 1986 - December 29 2016 Cushing, OK. Source: U.S. Energy Information Administration

of the rapidly changing economics of the fossil fuel industry. Media reports have referred to Canadian oil as being "a good choice for roller coaster fans."[3] Oil firms have adapted their investment and operating strategies to the "roller coaster." In a May 2015 interview one energy company executive stated that his company had been assuming $50 per barrel for crude oil for the benchmark WTI, but added that "We didn't believe $100 oil was going to last forever and we don't believe $50 will last forever."[4]

While the recent experience of oil sands development is particularly dramatic, parallels can be found in other resource extraction industries such as copper, potash, and gold. In-

[3]See for example, two 2012 Globe and Mail headlines: "Canadian oil: a good choice for roller coaster fans," (Nathan VanderKlippe in the Globe and Mail, August 24, 2012) and "Economics biggest threat to embattled oil sands," (Martin Mittelstaedt in the Globe and Mail, January 18, 2012).

[4]"Rachel Notley reaching out to the energy sector", Kyle Bakx, May 12, 2015, CBCnews, Business, http://www.cbc.ca/news/business/rachel-notley-reaching-out-to-energy-sector-1.3070996

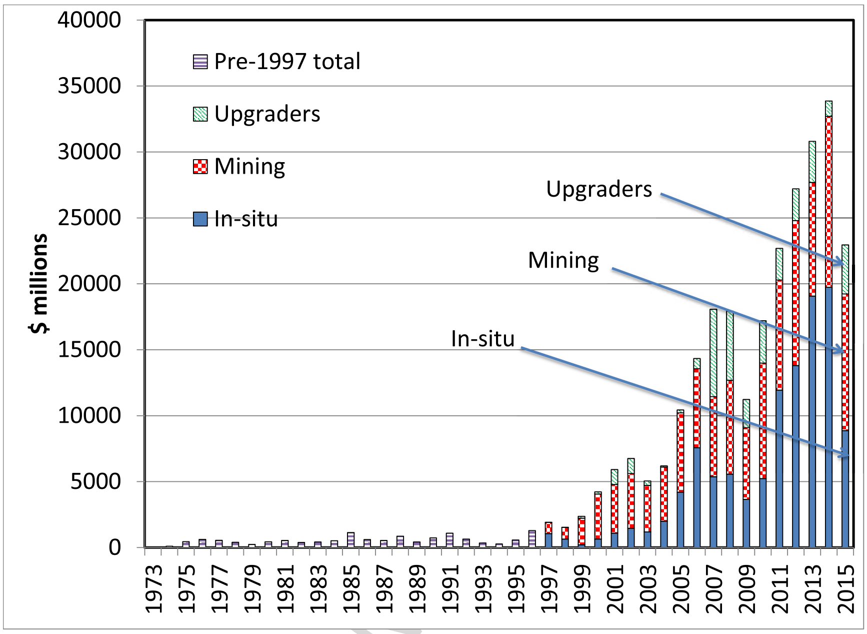


Figure 2: Alberta Oil Sands Capital Expenditures, 1973 - 2015, millions of C$. Data Source: Canadian Association of Petroleum Producers

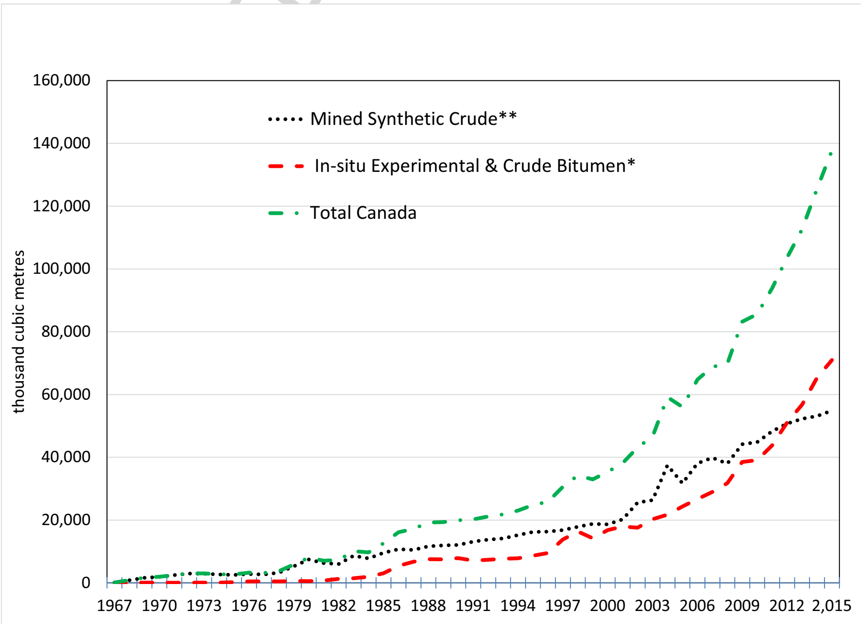


Figure 3: Canadian oil sands production, 1967 - 2015, Source: Canadian Association of Petroleum Producers.[Notes: *Effective 1985, experimental crude production excluded from the data. ** From upgraders integrated with mining projects]

dustries ramp up investment when prices are buoyant, with resultant environmental impacts and public concern. In this paper, we investigate the economics of non-renewable natural resource extraction taking account the boom and bust cycle of commodity prices. In particular we examine the optimal investment strategy when there is an expectation that in the future, prices may switch to a regime with starkly different dynamics than those observed currently. As has happened in Alberta, a sudden ramp up in resource investment and extraction may have environmental consequences which the public expects that regulators will be able to address. We seek to deepen our understanding of the optimal response of resource investment to uncertain commodity prices which provides the backdrop for devising appropriate environmental regulations. To this end, we model the decisions of a profit maximizing firm with the option to develop a non-renewable resource deposit, choosing the timing and pace of development, as well as the decision to produce the resource or shut down if prices become weak. To capture the boom and bust cycle typical of many commodities, the resource price is modelled as a regime switching process. The model is applied to a typical oils sands *in situ* project, but the analysis and results are relevant for other types of resource extraction operations. The model is used to solve for critical price levels at which it is optimal for a firm to invest in extraction, begin production, or shut down operations. The paper focuses on the impact of the prospect of regime shifts in commodity prices on optimal decisions and the pace of development. The paper also considers the effect on optimal decisions and production timing of the prospects of stricter environmental regulations in the form of a carbon tax .

The paper models optimal resource extraction as a stochastic optimal control problem using a real options approach. Brennan and Schwartz (1985), was one of the earlier papers showing how optimal policies for managing a natural resource can be derived using no-arbitrage arguments from the finance literature. Since the 1980's the literature using a real options approach for problems in natural resource and environmental economics has grown dramatically with diverse applications such as in forestry, fisheries, habitat protection, pollution control, and global warming.[5] Reviews of real options applications to resource problems are provided in Schwartz and Trigeorgis (2001) and Mezey and Conrad (2010).

Papers dealing specifically with optimal development of a non-renewable resource include Cortazar and Schwartz (1997) who use an options approach to value an undeveloped oil field. Slade (2001)contrasts the predictions of a real options model with decisions to open and close copper mines in Canada. Conrad and Kotani (2005) determine the optimal trigger price to begin drilling for oil in a wildlife preserve assuming stochastic oil prices, but also considering uncertainty in the amenity value that would be lost when drilling proceeds. Schwartz (1997) examines the impact of different models of the stochastic behaviour of commodity prices on the valuation and optimal decisions in resource extraction projects. Mason (2001) extends Brennan and Schwartz (1985) by modelling the decision to suspend or reactivate the extraction of a non-renewable resource when the finite resource stock is accounted for explicitly as an additional state variable. Mason examines the impact of the costs of suspension and reactivation (so-called switching costs) and observes a hysteresis or tendency for firms to continue with the status quo, whether currently operating or suspended. This is in the spirit of the work by Dixit (1989a,b, 1992); Dixit and Pindyck (1994). Almansour and Insley (2016) use a real options approach to examine optimal extraction of a non-renewable resource when price and costs are correlated stochastic processes. Muehlenbachs (2015) tests the goodness-of-fit for a real options model to actual firm behaviour in Alberta's oil industry. She focuses on the incentive to temporarily mothball oil field developments to avoid the reclamation costs associated with final abandonment. Kobari et al. (2014) use a real options approach to examine oil sands extraction in a multi-agent, non-strategic, setting.

[5] A recent application to global warming is Chesney et al. (forthcoming). Abdallah and Lasserre (2012) address the option to protect endangered species. Sarkar (2009) and Ewald et al. (2017) use an options approach in the context of fisheries. Chen and Insley (2012) is an application of a regime switching model to a tree harvesting problem.

Their paper assumes one-factor geometric mean reverting model of oil prices.

This paper contributes to the literature by solving for optimal resource investment and extraction decisions for a non-renewable resource assuming that price uncertainty can be characterized by a Markov-switching process - something not done previously in the literature to the best of our knowledge. With price and resource stock as state variables, we consider a multistage investment decision in which the owner must choose when to proceed through several phases of construction as well as whether to temporarily mothball a producing facility or permanently abandon it. The problem is specified as a Hamilton-Jacobi-Bellman (HJB) partial differential equation. An analytical solution is not available, hence a finite difference numerical approach is used to obtain the solution for a prototype oil sands investment problem. The HJB equation is solved for the case where there are two price regimes, and in each regime price follows a different mean reverting stochastic process. Parameter estimates for the price process are determined though a calibration procedure using oil futures prices. The paper does not focus on the econometric issues involved in obtaining the best parameter estimates. Rather the focus is on examining the impact of regime switching on the optimal decision.

Our findings show that decision makers who anticipate random regime shifts in resource prices will behave differently than those who expect the pricing environment to remain in the current cycle for many years. In particular the timing of investment and extraction decisions are different, implying that regime shifts are important in analyzing optimal extraction decisions. We examine the pattern of critical prices as project construction proceeds through various stages. Each stage of construction may be viewed as an option to take the next step towards having a producing property. It is shown that under reasonable assumptions critical prices to exercise the option to move to the next construction phase start low and then rise as construction proceeds, implying a lower optimal price to begin construction than to complete the project. This will be of interest to environmental regulators to the extent that different phases of the project have more or less severe environmental consequences. In the case of oil sands and other large mining projects, the early construction phases may have significant environmental consequences as sites are made ready for extraction. This underlines the need to have the regulatory framework in place to deal with a surge in interest by firms to undertake resource extraction projects. A change in environmental regulations may affect the pace of development as well as the timing of abandonment. Under reasonable assumptions we show that a gradually increasing carbon tax speeds up current oil sands development, whereas a sudden increase in the carbon tax slows down the pace of development and increases the possibility of the abandonment of a project prior to exhaustion of reserves.

The next section of the paper describes alternate ways of modelling oil prices and presents the regime switching model. The resource valuation model including the solution approach is described in Section 3. Section 4 explains the methodology used to calibrate the parameters of price process. A description of the oil sands example and analysis of the results is presented in Section 5. A summary and concluding remarks are given in Section 6.

2. The stochastic oil price process

Considerable effort has been made in the literature to determine the best models of commodity prices. The criteria for judging what is "best" depends on the goal - whether pricing commodity based derivatives, matching the term structure of futures prices, valuing long term investments, or other objectives. In this paper we are examining the optimal control of resource extraction over the long term. In this context, the price model should capture the long run dynamics of oil prices, but should also be parsimonious so that interpretation of the optimal control is not problematic.

For convenience, many papers adopt a simple process geometric Brownian motion (GBM) to describe uncertain commodity prices such as in the much cited paper by Brennan and Schwartz (1985) who used GBM model for oil prices. However as is noted by Schwartz (1997) among others, economic logic suggests that commodity prices should tend to some long run mean determined by the marginal cost of new production and the price of substitutes. In addition the volatility of futures prices tends to decrease with maturity, whereas a simple GBM process implies that futures prices will have constant volatility.[6] Two mean reverting processes which have been used in the literature include:

$$dP = \eta(\bar{P} - P)dt + \sigma P dz \tag{1}$$

and

$$dP = \eta(\mu - log(P))Pdt + \sigma P dz \tag{2}$$

where P denotes price; η and σ represent the speed of mean reversion and price volatility, respectively; and $\bar{P}$ and μ are the long run equilibrium levels of price and the log of price, respectively. Equation (1) has been used in various contexts such as in Insley and Rollins (2005) to model timber prices and in Chen and Forsyth (2007) to model natural gas prices. Schwartz (1997)uses Equation (2)to model oil, copper and gold prices. Neither of these models is fully satisfactory in terms of their ability to describe the behaviour of commodity futures prices. Although the implied volatility of futures prices decreases with maturity, which is desirable, volatility tends to zero for very long maturities, which is not a phenomenon observed in practice (Chen and Insley, 2012). A better description of commodity prices can be obtained by including additional stochastic factors. Schwartz (1997) compared two and three factor models with the one factor model of Equation (2). The two and three factor models clearly outperformed the one factor model in terms of modelling the term structure of futures prices as well as the term structure of volatilities for copper and oil.

Another strand of the literature allows the variance of the stochastic process to change

[6]See Chen and Insley (2012) for further discussion and references.

at discrete points in time or continuously. For example Larsson and Nossman (2011) model oil prices with volatility as a stochastic process with jumps and use a Markov chain Monte Carlo method to estimate parameters using WTI crude spot prices. The estimates obtained are consistent with the spot price under the $\mathbb{P}$-measure. Note that if the goal is to price options or analyze investment decisions, it is desirable to estimate risk adjusted parameters under the $\mathbb{Q}$ measure.[7]

A regime switching model provides an alternate approach to capturing non-constant drift and volatility terms for the stochastic process followed by oil prices. First described by Hamilton (1989), it has intuitive appeal in that the boom and bust periods of commodity prices may be thought of as different price regimes each characterized by a unique stochastic process. Regime switching has been considered in the context of macroeconomic cycles such as in Hamilton (1989) and Lam (1990). Guo et al. (2005) notes that macroeconomic business cycle regimes may potentially have significant impacts on firms' choices, and that "... despite these potential effects we still know very little about the relation between regime shifts and investment decisions."

Regime switching models have been used by several authors to capture the dynamics of electricity prices. An overview can be found in Janczura and Weron (2010) and Niu and Insley (2016). Chen and Forsyth (2010) use a regime switching model of natural gas prices to examine optimal decisions in a natural gas storage operation. Chen and Insley (2012) model lumber prices as a regime switching process to examine optimal tree harvesting decisions.

In a regime switching model, different regimes are defined which can accommodate dif-

[7](Björk, 2003) provides details on the relationship between $\mathbb{Q}$-measure and $\mathbb{P}$-measure parameters.

ferent specifications of price behaviour. A general regime switching process is given as:

$$
\begin{aligned}
dP &= a^j(P,t)dt + b^j(P,t)dz + \sum_{l=1, l\neq j}^{J} P(\xi^{jl}-1)dX_{jl} \\
j &= 1,...,J,\ l = 1,...,J
\end{aligned}
\tag{3}
$$

j and l refer to regimes and there are J regimes with j being the current regime. $a(P,t)$ and $b(P,t)$ represent known functions and dz is the increment of a Wiener process. When a regime switch occurs, the price level jumps from P to $\xi_{jl}P$. The term dX_{jl} governs the transition between j and l:

$$
dX_{jl} = \begin{cases} 1 & \text{with probability } \lambda^{jl}dt \\ 0 & \text{with probability } 1-\lambda^{jl}dt \end{cases}
\tag{4}
$$

For simplicity in this paper we make the assumption that there are only two possible regimes, and in each regime price follows an independent stochastic process as follows:

$$
\begin{aligned}
dP &= \eta^j(\bar{P}^j - P)dt + \sigma^j P dz \\
j &= 1,2;
\end{aligned}
\tag{5}
$$

where η^j is the speed of mean reversion in regime j, $\bar{P}^j$ is the long run price level in regime j, and σ^j is the volatility in regime j. Regime switching is governed by Poisson process dX_{jl} specified in Equation (4). It will be noted that we do not include a jump term which allows price to jump suddenly to a new level when a regime change occurs. This is done to simplify parameter calibration. The transition to a new regime entails only new drift and volatility terms. However if the speed of mean reversion is quite high, then the switch to a new regime will cause a change in the price level, as price is pulled towards its new long run mean.

The regime switching price model chosen here is similar to the one used by Chen and

Forsyth (2010) to analyze a natural gas storage problem. In that paper, natural gas prices were assumed to follow a process similar to Equation (5), except that a seasonality component was also included. Note that seasonality has not typically been included in models of oil prices (Schwartz, 1997; Borovkova, 2006). This paper is concerned with long run investment decisions in oil sands, and seasonality would not be important for such decisions.

The parameters of Equations (4) and (5) are estimated by calibrating to oil futures prices. We estimate the risk adjusted parameter values which reflect market expectations about future prices.[8] The calibration procedure and estimated parameter values are presented in Section 4.

3. Resource Valuation Model

3.1. Specifying the Decision Problem

We model the optimal decision of a firm regarding when to invest in the extraction of a non-renewable resource, which is an oil sands project for the purposes of this paper. The project has significant capital costs and construction takes several years. The firm's decision is taken in the context of uncertain prices characterized by Equations (4) and (5). The firm's objective is to maximize the value of the resource asset by optimally choosing an extraction path over time, as well as determining the optimal timing for construction, production, temporarily mothballing the operation, reactivating from a mothballed state, and finally abandoning the property. Let $V_m^j(P, S, t) \equiv V(P, S, t, \delta_m, j)$ be the market value of cash flows from the resource extraction project where:

- P is the resource price, $P \in [0, \infty]$
- S is the size of the resource stock, $S \in [0, S_0]$, where S_0 is the original size of the reserve

[8] Note that the Girsanov transformation could be used to determine real world parameters corresponding to the risk neutral parameters. However this requires additional calibration.

- t is time, $t \in [t_0, T]$
- δ_m represents the project stage, $m = 1, ..., M$, i.e. under construction, producing, mothballed, or abandoned
- j is the regime, $j = 1, ..., J$.

The firm chooses the extraction rate, R^j_m, which depends on the current regime j and operating stage m, and at certain discrete points in time is able to switch to a different operating stage by incurring a cost. Let $Z(S)$ represent the admissible set for R^j_m.

$$
\begin{aligned}
& Z(S) \in [0, R_{\max}],\ S \neq 0 \\
& Z(S) = 0,\ S = 0
\end{aligned} \tag{6}
$$

The change in the stock of the resource is then $dS = -R^j_m dt$.

Let t_n, $n = 1, ..., N$ be discrete decision dates and the admissible set for the project stage, δ_m, be $Y = [\delta^j_1, ..., \delta^j_M]$ where changes in δ^j_m can only occur at discrete times t_n. In other words, if δ^j_m is the optimal choice at $t = t_n$, then $\delta^j = \delta^j_m$ for $t_{n-1} \leq t \leq t_n$. Note that in principal, we could find the numerical solution for the case where we allow $\Delta t = t_n - t_{n-1} \to 0$. In other words, we allow for an optimal decision at each discrete timestep. This would converge to the impulse control formulation of the problem (see, e.g. Chen and Forsyth (2008)). However, this becomes computationally infeasible if we want to include decision making during partial project completion. This is probably also unrealistic: firms do not make stop - go decisions about large capital expenditures every day.

When the project is operational a cash flow, $\pi^j_m(t)$, is earned as follows:

$$
\pi^j_m(t) = R^j_m(P, S, t)\Big(P(t) - c_v\Big) - c_f - \text{taxes} \tag{7}
$$

c_v is per unit variable cost and c_f is per unit fixed cost.

The value of the asset, V, is determined by maximizing the expected present value of profits from the initial period t_0 before construction has begun through to time $t = T$, when the project must be permanently shut down. Note that the timing of project shut-down is determined endogenously in the model and will depend on the stochastic price of oil and level of reserves. The permanent closure of the project can be no later than at time T and may be due to the exhaustion of reserves or some other reason such as the end of a lease. T can be set arbitrarily large to mimic an infinite horizon problem.

We let p and s denote the price and stock respectively at a particular moment in time. The market value of the project in regime j and stage m is $V_m^j(p, s, t)$ where

$$
\begin{aligned}
&V_m^j(p,s,t) = \max_{R,\delta_m} \mathrm{E}^Q \left\{ \int_{t_0}^{T} e^{-rt} \left[\pi_m^j\right] dt \mid P(t) = p, S(t) = s \right\}, \\
&m = 1, ..., M;\ j = 1, ..., J \\
&\text{subject to } \int_{t_0}^{T} R(:,t)dt \leq S_0.
\end{aligned}
$$

The constraint states that total production of the resource cannot exceed the initial stock in place.

We use standard contingent claims arguments to derive a system of PDE's describing V in project stage δ_m between decision dates. Let $t_n^+ = t_n + \epsilon$ and $t_n^- = t_n - \epsilon$ where $\epsilon > 0,\ \epsilon \to 0$. Then between decision dates we have:

$$
\frac{\partial V_m^j}{\partial t} = \max_{R \in Z(S)} \left\{ -\frac{1}{2} b^j(p,t)^2 \frac{\partial^2 V_m^j}{\partial p^2} - a^j(p,t) \frac{\partial V_m^j}{\partial p} + R_m^j \frac{\partial V_m^j}{\partial s} - \pi_m^j(t) + \sum_{l=1, l \neq j}^{J} \lambda^{jl} (V_m^l - V_m^j) - r V_m^j \right\} \tag{8}
$$

$$
j = 1, ..., J;\ m = 1, ..., M
$$

where $a^j(p,t)$ is the risk adjusted drift rate conditional on $P(t) = p$ and λ^{jl} is the risk adjusted transition to regime l from regime j. For our chosen price process $a^j(p,t) \equiv \eta^j(\bar{P}^j - p)$ and $b^j(p,t) \equiv \sigma_j p$.

At decision dates, the decision maker will check to see if it is optimal to switch to a different operating stage. There are M operating stages. Let $C_{\bar{m}m}$ denote the cost of switching from the current stage $\bar{m}$ to another stage m. Let $t = t^-$ denote the moment before a decision is taken and $t = t^+$ the moment after a decision. The value of the asset is the maximum of the values at all possible stages, m, net of the cost of getting there.

$$V(t^-, \delta_{\bar{m}}) = \max\left\{V(t^+, \delta_1) - C_{\bar{m}1},\ \ldots, V(t^+, \delta_{\bar{m}}) - C_{\bar{m}\bar{m}},\ \ldots, V(t^+, \delta_M) - C_{\bar{m}M}\right\} \tag{9}$$

Note that it is assumed there are fixed costs in each stage, so that if the firm chooses to remain in the current stage $\bar{m}$, it will incur a cost to do so. Of course, it is also possible to have $C_{\bar{m},\bar{m}} = 0$, i.e. it is costless to remain in the current stage. However we choose to include a small positive cost to reflect the costs of maintaining the site and equipment in the event that construction may be restarted in the future.

For computational purposes Equation (8) is solved over the finite domains $P \in [0, P_{\max}]$ and $S \in [0, S_0]$. For convenience we write out Equation (8) substituting for a^j and b^j. In addition, using the usual dynamic programming technique we must solve backwards from the final time $t = T$ to the initial period $t = t_0$. It is convenient to define $\tau = T - t$ as the time remaining in the life of the asset. We then solve from $\tau = 0$ to $\tau = T$. Below Equation (8) is specified in terms of τ. Note that we also show the maximization with respect to R only for those terms that contain R.

$$\frac{\partial V_m^j}{\partial \tau} = \frac{1}{2}\sigma_j p^2 \frac{\partial^2 V_m^j}{\partial p^2} + \eta^j(\bar{P}^j - p)\frac{\partial V_m^j}{\partial p} + \max_{R \in Z(S)}\left\{\pi_m^j - R_m^j \frac{\partial V_m^j}{\partial s}\right\} + \sum_{l=1, l\neq j}^{J} \lambda^{jl}(V_m^l - V_m^j) - rV_m^j \tag{10}$$

$$j = 1, \ldots, J;\ m = 1, \ldots, M.$$

3.2. Boundary Conditions

Boundary conditions must be specified to fully characterize the resource valuation problem. Taking the limit of Equation (10) as $P \to 0$ gives:

$$\frac{\partial V_m^j}{\partial \tau} = \eta^j \bar{P}^j \frac{\partial V_m^j}{\partial p} + \max_{R \in Z(S)} \left\{ \pi_m^j - R_m^j \frac{\partial V_m^j}{\partial s} \right\} + \sum_{l=1, l \neq j}^{J} \lambda^{jl} (V_m^l - V_m^j) - r V_m^j \tag{11}$$

$$j = 1, ..., J; m = 1, ..., M.$$

At $P = 0$, the PDE reduces to first order hyperbolic, with outgoing characteristics. This means that we can simply solve the PDE (11) at $P = 0$, and no boundary condition is required. The PDE itself supplies the necessary boundary information. No information from outside the computational domain is needed.[9]

As $P \to Pmax$ we assume that $\frac{\partial^2 V_m^j}{\partial P^2} \to 0$, which is a common assumption used in the literature [10]. Essentially, this is equivalent to assuming that $V_m^j \to A(\tau) + B(\tau)P$ as $P \to \infty$, for some (unknown) functions $A(\tau), B(\tau)$. In other words, the value of the project is linear in P as the price of oil becomes very large. Equation (10) then becomes

$$\frac{\partial V_m^j}{\partial \tau} = [\eta^j (\bar{P}^j - p)] \bigg|_{p \to Pmax} \frac{\partial V_m^j}{\partial p} + \max_{R \in Z(S)} \left\{ \pi_m^j - R_m^j \frac{\partial V_m^j}{\partial s} \right\} + \sum_{l=1, l \neq j}^{J} \lambda^{jl} (V_m^l - V_m^j) - r V_m^j \tag{12}$$

$$j = 1, ..., J; m = 1, ..., M.$$

No further specifications are needed as we will always have $Pmax > \bar{P}$. This implies that along the boundary $P = Pmax$. As discussed above, this means that there are outgoing characteristics and no information outside the domain of P is required to compute the

[9] See Duffy (2006) for a discussion of boundary conditions and finance difference methods.

[10] See for example (Wilmott, 1998, chapter 46)

solution.

The domain of the resource stock is $S \in [0, S_0]$ and S is depleted by production, R_m^j:

$$S(t) = S_0 - \int_{t_0}^{t_n} R_m^j(t)dt \tag{13}$$

Z is the admissible set of R defined in Equation (6). As $S(t) \to 0$, the admissible set of R collapses to 0. We set $V = -D$ where D represents the present value of required restoration costs. Once reserves are depleted it is a regulatory requirement that restoration of the site must be undertaken.

For $S = S_0$, we solve Equation (10) at this boundary. The PDE reduces to first order hyperbolic at this boundary, and as in the cases described above, the PDE provides the information needed at the boundary, since there are outgoing characteristics in the S direction.

When $\tau = 0$ $(t = T)$, we assume $V(P, S, \tau = 0) = 0$.

3.3. Solution approach

Equations (8) and (9) represent a stochastic optimal control problem which must be solved using numerical methods. Define $\mathcal{L}V$ as a differential operator where:

$$\mathcal{L}V_m^j = \frac{1}{2}\sigma^j p^2 \frac{\partial^2 V_m^j}{\partial p^2} + \eta^j(\bar{P}^j - p)\frac{\partial V_m^j}{\partial p} + \sum_{l=1, l\neq j}^{J} \lambda^{jl}(V_m^l - V_m^j) - rV_m^j \tag{14}$$

Using $\mathcal{L}V_m^j$ as defined above, the partial differential equation, Equation (10), can be written as:

$$\frac{\partial V_m^j}{\partial \tau} - \max_{R \in Z(S)} \left[\pi_m^j - R_m^j \frac{\partial V_m^j}{\partial s}\right] - \mathcal{L}V = 0;\ j = 1, ..., J;\ m = 1, ..., M. \tag{15}$$

LV in Equation (15) is discretized using a standard finite difference approach. The other terms in the equation are discretized using a semi-Lagrangian scheme. Consider the path, $\mathcal{S}$

defined by the ordinary differential equation:

$$\frac{ds}{d\tau} = -R \tag{16}$$

Use Equation (16) to write two terms from Equation (15), $\frac{\partial V_m^j}{\partial \tau} + R_m^j \frac{\partial V_m^j}{\partial s}$, as a Lagrangian directional derivative:

$$\frac{DV_m^j}{D\tau} = \frac{\partial V_m^j}{\partial \tau} - \frac{\partial V_m^j}{\partial s}\frac{ds}{d\tau}. \tag{17}$$

Equation (15) can then be rewritten as

$$\max_{R \in Z(S)} \left[\frac{DV_m^j}{D\tau} - \pi_m^j \right] - \mathcal{L}V_m^j = 0;\ j = 1, ..., J;\ m = 1, ..., M. \tag{18}$$

A semi-Lagrangian discretization is implemented for Equation (18) as described in Chen and Forsyth (2007, 2010). Further details are provided in Appendix A.

Within the admissible set $R \in Z(S)$ we define a grid $[0, ..., R_{\max}]$ over which we check for the choice of R that maximizes Equation (18) at each time step. Given the nature of the revenue and cost functions used in this example, the optimal choice of R turns out to be a bang-bang solution - either 0 or $Rmax$.

4. Calibrating the parameters of the price process

4.1. Methodology

We use oil futures prices to calibrate the parameters of Equations (4) and (5) (except for the σ^j). The process is similar to that described in Chen and Forsyth (2010) and Chen and Insley (2012). Let $F^j(P, t, T)$ denote the futures price in regime j at time t with delivery at T while the spot price resides at P. (This will be shortened to F^j when there is no ambiguity.) On each observation day, t, there are futures contracts with a variety of different maturity dates, T. The futures price equals the expected value of the spot price in the risk neutral world. We set $J = 2$, assuming that there are two possible regimes in order to keep the computational complexity to a manageable level.

$$\begin{aligned} F^j(p,t,T) &= E^Q[P(T)|P(t)=p, J_t=j] \\ j &= 1,2. \end{aligned}$$

where E^Q refers to the expectation in the risk neutral world and J_t refers to the regime in period t. Applying Ito's lemma results in two coupled partial differential equations for the futures price, one for each regime:

$$(F^j)_t + \eta^j(\bar{P}^j - P)(F^j)_P + \frac{1}{2}(\sigma^j)^2 P^2 (F^j)_{PP} + \lambda_{jl}(F^l - F^j) = 0,\ j = 1,2. \tag{19}$$

with boundary condition $F^j(P,T,T) = P$. The solution of these coupled pde's is known to have the form

$$F^j(P,t,T) = a^j(t,T) + b^j(t,T)P \tag{20}$$

Substituting this solution into Equation (19) yields a system of ordinary differential equations:

$$\begin{aligned} (a^j)_t + \lambda^{jl}(a^l - a^j) + \eta^j \bar{P}^j b^j = 0 \\ (b^j)_t - (\eta^j + \lambda^{jl})(b^j) + \lambda^{jl} b^l = 0,\ j = 1,2. \end{aligned} \tag{21}$$

$(a^j)_t \equiv \partial(a^j)/\partial t$ and $b(s_t)_t \equiv \partial b(s,t)/\partial t$. Substituting boundary condition $F^j(P,T,T) = P$ into Equation (20) gives $a^j(t=T,T) = 0$; $b^j(t=T,T) = 1$.

Taking the matrix differential of Equation (21) gives:

$$\frac{d}{dt}[a^1\ a^2\ b^1\ b^2]' = A[a^1\ a^2\ b^1\ b^2]' \tag{22}$$

The solution to Equation (22) is:

$$[a^1\ a^2\ b^1\ b^2]' = e^{At}[0\ 1\ 0\ 1]' \tag{23}$$

where, e^{At} is the matrix exponential, and

$$A = \begin{bmatrix} -\lambda^{12} & \eta^1\bar{P}^1 & \lambda^{12} & 0 \\ 0 & -(\eta^1+\lambda^{12}) & 0 & \lambda^{12} \\ \lambda^{12} & 0 & -\lambda^{12} & \eta^2\bar{P}^2 \\ 0 & \lambda^{12} & 0 & -(\eta^2+\lambda^{21}) \end{bmatrix}. \tag{24}$$

Let θ denote the suite of parameters to be estimated: $\theta = \{\eta^j, \bar{P}^j, \lambda^{jl} \mid j, l \in \{0,1\}\}$. In addition the current regime, $j(t)$, must be estimated. σ^j is not included in θ as it must be estimated separately from the other parameters. This follows from the observation that σ^j does not appear in Equation (21), implying that for this particular price process the futures price at any time t does not depend on spot price volatilities. Determination of the σ^j for each regime is discussed below.

The calibration is carried out by finding the parameter values which minimize the ℓ_2 norm error (root mean square error) between model-implied futures prices and actual futures prices.

$$\min_{\theta, j(t)} \sum_t \sum_T (\hat{F}(J(t), P(t), t, T; \theta) - F(t,T))^2 \tag{25}$$

$$\text{subject to } \eta_{\min} \le \eta \le \eta_{\max},\ \bar{P}_{\min} \le \bar{P} \le \bar{P}_{\max},\ \lambda^{ij}_{\min} \le \lambda^{ij} \le \lambda^{ij}_{\max} \tag{26}$$

where $F(t,T)$ is the market futures price on observation day t with maturity T and $\hat{F}(J(t), P(t), t, T; \theta)$ is the corresponding model implied futures price calculated from Equations (20), (23), and (24) above. Equation (26) is a constrained non-linear optimization problem with possibly

	Regime 1	Regime 2	lower bound	upper bound
η^j	0.44	1.05	0.01	3
$\bar{P}^j(US\$/bbl)$	75	30	0	150
λ^{jl}	0.26	0.28	0.02	0.6
σ	0.23	0.44		

Table 1: Case 1 (base case) parameter estimates. $dP = \eta^j(\bar{P}^j - P)dt + \sigma^j P dz, j = 1, 2$. Note that $\bar{P}$ is in U.S.$ and refers to West Texas Intermediate. Parameters have been annualized.

many local minimums. In order to get meaningful results we must impose economically sensible limits on the ranges of possible parameter values.

It is known that for Ito processes such as Equation (5), the volatilities, σ^j, are the same under the $\mathbb{P}$-measure as under the $\mathbb{Q}$-measure. It is therefore possible to use spot prices to estimate values for the σ^j. In fact, we use the nearest month futures price as a proxy for the spot price, as is common in the literature. This ensures consistency of futures and spot prices. For this paper we use the methodology of Perlin (2012) to estimate Markov state switching models.[11]

4.2. Data description and calibration results

The calibration was carried out using average monthly prices for WTI futures contracts on the New York Mercantile exchange. The prices were converted to 2016 dollars using the U.S. GDP deflator.[12] Crude oil futures are available for nine years forward: consecutive months are listed for the current year and the next five years; in addition, the June and December contract months are listed beyond the sixth year. The calibration is done for all available contract maturities from 2 months through 9 years which amounts to 107 different contract maturities. Data used is from January 1995 to December 2016.

[11] A alternative approach would be to estimate σ^j using data for the prices of options on oil futures as is done in Chen and Forsyth (2010).

[12] Data was obtained from Datastream. Daily data was converted to monthly averages. This was deemed appropriate since the economic questions of interest relate to long run investment and production decisions, and there is no need to consider higher frequency data. The data was put in constant dollar terms so that the estimated parameters would reflect the characteristics of real oil prices.

Results of the calibration are given in Table 1. For 14,842 data points the minimized ℓ_2 norm error is 1,352,910 implying an average error of \$9.55 [13]. The table also reports the upper and lower bounds imposed on the optimization. The same bounds were imposed in each regime. There is no expectation that there is a unique solution to the least squares minimization procedure. The bounds are imposed to help ensure an economically reasonable solution is obtained from the optimization process. It is well known that $\mathbb{Q}$-measure calibration is very sensitive to noisy data. The calibrated parameters seem to be economically reasonable, but should not be considered definitive. Rather they are illustrative of the effects of a highly uncertain environment.

The results show two distinct regimes. These parameter estimates are in the $\mathbb{Q}$-measure and reflect the expectations of the market including a market price of risk. Regime 1 has a long run mean of price of US\$75 with a mean reversion speed of 0.44, which, ignoring volatility, roughly implies an expected time to revert to the mean of 2.3 years. Regime 2 has a higher speed of mean reversion of 1.06 to a long run mean of US\$30 per barrel. The λ's are close in value. The expected time to remain in regime j is $1/\lambda_j$, which is a bit over 3.5 years. In what follows we will on occasion refer to Regime 1 as the 'high price regime' reflecting the higher long run mean price compared to Regime 2, which we will refer to as the 'low price regime'.

The volatility estimates, obtained using Perlin (2012), show one regime with a significantly lower volatility. One complication is in assigning the volatilities estimated in the $\mathbb{P}$-measure to the regimes determined through the $\mathbb{Q}$-measure calibration, which amounts to a simple string matching problem. This is done by observing which of the volatilities is assigned to each of the months of the estimation period by the $\mathbb{P}$-measure calibration. We then compare this to the $\mathbb{Q}$-measure estimate of regimes by month, and assign the volatility

[13] $\sqrt{(1,352,910/14,842)} = 8.85$

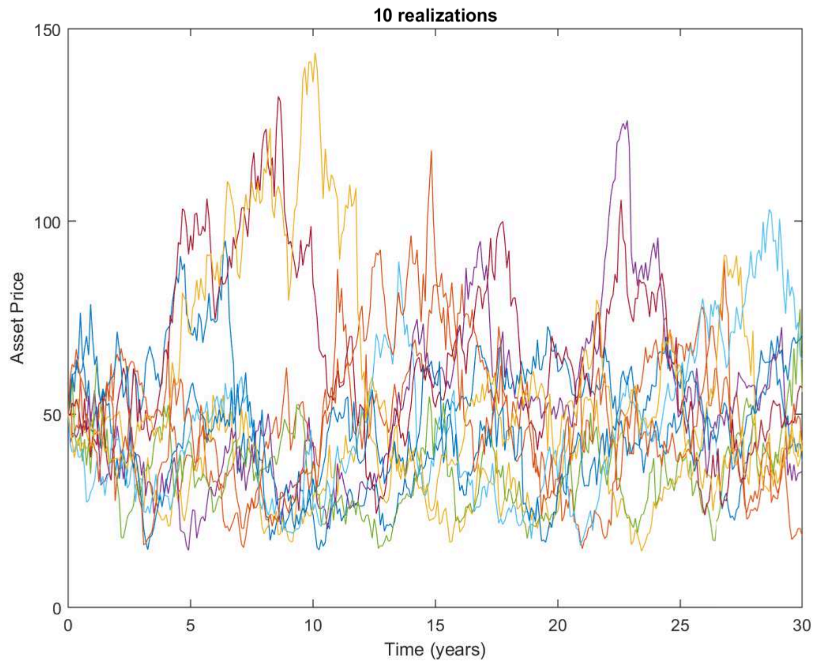


Figure 4: Simulation of base case regime switching price process, U.S. $/barrel for WTI

to each regime that best matches the time series profile.

As an aid to visualization, we show in Figure 4 a simulation of 10 realizations of the process with a starting price in Regime 1 of US$50. The result is a highly volatile price that does not spend time resting at either of the long run means.

5. Oil extraction decision problem

5.1. Project specification

We examine the decision to build and operate an oil sands *in situ* extraction project. Mining and *in situ* are the two methods currently used to extract bitumen[14] from Alberta's oil sands with *in situ* used for deposits too deep to be mined. It is estimated that 80% of

[14]Bitumen is oil that is too heavy or thick to flow or be pumped, at ambient temperatures, without being diluted or heated.

Project type	In situ, SAGD
Production capacity	30,000 bbl/day
Average capacity factor	75%
Reserves	250 million barrels
Production life length	30 years
Construction cost	C$1090.8 million over three years
Variable operating costs (non-energy)	C$6.54 per barrel
Variable operating costs (natural gas)	C$4.79 per barrel
Variable operating costs (electricity)	C$0.55 per barrel
Fixed operating costs	C$44 million per year
Fixed cost (sustaining capital)	C $55 million per year
Abandonment and reclamation	C$21.8 million (2% of capital costs)
Cost to mothball and reactivate	C$ 10 million
Federal corporate income tax	15%
Provincial corporate income tax	10%
Carbon tax per ton CO2e	C$30 in year 1, rising at 2% per year thereafter

Table 2: Details of the prototype *in situ* project.

Alberta's remaining recoverable bitumen is suited to *in situ* extraction involving steam or solvent injection through horizontal or vertical wells (Millington et al., 2012).

The characteristics of the prototype oil sands project are summarized in Table 2. Production capacity, production life, and construction and operating costs are taken from a report produced by the Canadian Energy Research Institute (CERI) (Millington and Murillo, 2015). CERI's estimate of total construction costs of C$1090.8 million are assumed to be spread over three years. Variable costs are comprised of natural gas, electricity and non-energy costs. The CERI assumption is that a project of this magnitude will require 35,910 GJ per day of natural gas and 300 MWH per day of electricity. We assume that the cost of natural gas remains constant in real terms at C$3/GJ, which amounts to C$4.79 per barrel of bitumen production from this project. The cost of electricity is assumed to remain at C$41.49/MWH in real terms, which is C$0.55 per barrel of bitumen produced. Of course, both electricity and natural gas prices could be modelled as separate stochastic factors. However this is not

WTI price C$/barrel	Gross revenue royalty rate	Net revenue royalty rate
$P < 55$	1 %	25 %
$55 \leq P \leq 120$	Increases linearly	Increases linearly
$P > 120$	9 %	40 %

Table 3: Alberta's royalty rates for oil sands production, (Government of Alberta, 2007)

the focus of the paper, and so we make these simplifying assumptions.[15]

Total non-energy costs operating costs are given by CERI as \$97.7 million annually. There is little information available on the fixed/variable split for operating costs. Based on personal communications with an industry representative, we have allocated 55% to variable costs and 45% to fixed costs. This implies non-energy variable costs are \$53.7 million or \$6.34 per barrel of production. The remaining portion of \$44 million is considered fixed operating costs. Another large fixed cost reported by CERI is C\$44 million for sustaining capital, which is spending required to maintain operations at existing levels.

We adopt the CERI assumption for the cost of abandonment and reclamation of 2% of capital costs. Costs for mothballing and reactivation are the author's assumption. There have been no reports of mothballing of oil sands projects, despite the recent downturn in prices, which implies the costs of doing so are significant.

Firms producing Alberta oil must pay royalties to the provincial government. Royalty rates differ depending on whether or not a firm has recovered the allowed project costs. Prior to the payout date, royalties are paid on gross revenues[16] at the gross revenue royalty rates shown in Table 3. After payout has been achieved royalties are the greater of the gross revenue royalty or the net revenue royalty based on the net revenue royalty rate shown in

[15] See Almansour and Insley (2016) for a study of the relationship between oil and natural gas prices and the impact of this relationship on the economics of an oil sands operation.

[16] Gross revenue is defined as the revenue collected from the sale of oil sands products (or the equivalent fair market value) less costs of any diluents contained in any blended bitumen sold. Allowed costs are those incurred by the project operator to carry out operations, and to recover, obtain, process, transport, or market oil sands products recovered, as well as the costs of compliance with environmental regulations and with termination of a project, abandonment and reclamation of a project site. (Millington et al., 2012)

the same table. The implication is that the royalty rate is a path dependent variable in that the date of payout is dependent on the stochastic oil price, making the calculation of the post-payout royalty non-trivial. For simplicity, we have used the pre-payout royalty rate in our analysis.

For the base case a carbon tax of C$30 per tonne of CO_2e[17] is applied in the first year, increasing at 2% per year in real terms over the 30 year time frame of the analysis. This is the tax recommended by Alberta's Climate Change Advisory Panel and which is currently being implemented.[18] For large industrial facilities like oil sands operations, the tax is just one aspect of the so-called Carbon Competitiveness Regulation (CCR). Under the CCR the carbon tax is paid only on emissions exceeding a particular allowance or output based allocation which reflects top-quartile performance on emissions. This implies that new projects with low emissions will have lower compliance cost than projects with higher emissions. In the analysis which follows, we consider several sensitivities on the carbon tax. Note that a carbon tax of $30 per tonnes converts to $2.34 per barrel of bitumen assuming 78 kilograms of CO_2 are created in the production of 1 barrel of bitumen.[19]

The oil price model is calibrated using data on the price of futures for WTI on the NYMEX exchange in $U.S./barrel. The price paid for bitumen in Alberta is at a discount to the WTI price due to its lower quality, and more recently due to the lack of pipeline capacity. Transportation and the exchange rate also contribute to the differential, which can be highly volatile. In this paper we fix the price of bitumen in the field in Canadian dollars at 83% of the price of WTI crude in US dollars. This reflects the average ratio over the past ten years of Western Canada Select at Hardisty in $C/bbl to WTI in U.S.$/bbl.[20] The

[17] carbon dioxide equivalent

[18] Climate Leadership Report to Minister, November 2015, *https://www.alberta.ca/climate-leadership-discussion.aspx*.

[19] This estimate is from Israel (2016).

[20] See the Canadian Association of Petroleum Producers, Statistical Handbook, *http://www.capp.ca/publications-and-statistics/statistics/statistical-handbook*.

explicit modelling of the price of bitumen relative to the price of WTI is beyond the scope of this paper. However we undertake sensitivity analyses in relation to the long run mean price of oil as well as the volatility, which can capture to some extent the impact of these additional risk factors. Similarly the Canadian-U.S. dollar exchange rate could be modelled as additional stochastic factor but this is beyond the scope of this paper.

We model our prototype project as having the option to proceed through six stages, with the decision maker choosing the optimal time to move from one stage to the next. It is assumed that firms check annually to determine whether to switch from one stage to the next. The stages are as follows.

- Stage 1: Before construction begins
- Stage 2: Project 1/3 complete
- Stage 3: Project 2/3 complete
- Stage 4: Project 100 % complete and in full operation
- Stage 5: Project is temporarily mothballed
- Stage 6: Project abandoned

The decisions to move from Stages 1 to 2, 2 to 3, or 3 to 4 each require spending 1/3 of the total constructions costs. The decision maker has the option to postpone moving through these construction stages, but staying longer than a year in any phase once construction has begun (i.e. stages 2 and 3) is assumed to incur extra costs of C$1 million per year. Moving to Stage 4 also requires spending fixed and variable operating costs. We include the option to temporarily mothball production at an assumed cost of C$10 million as well as the option to reactivate for another C$10 million. When the project is mothballed it is assumed that only the sustaining capital is incurred. Finally, there is the option to abandon the project at a cost of 2% of construction costs.

5.2. Results Analysis

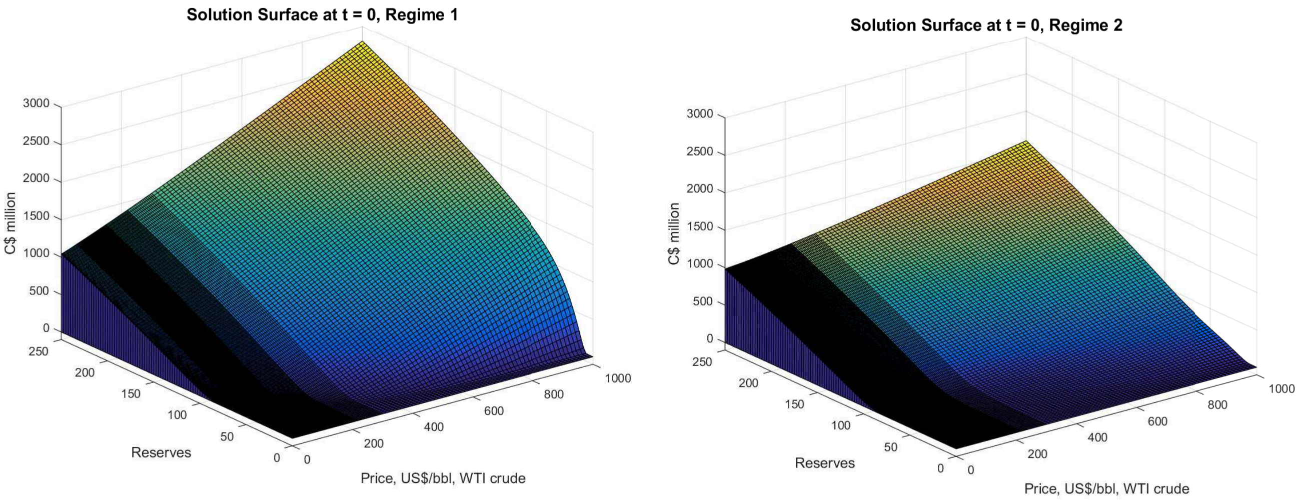
5.2.1. Case 1 (base case)

We determine the optimal decisions and asset value by solving the HJB equation (Equation 18) using the project specifications and costs detailed in Tables 2 and 3 and parameter values of Table 1. In discussing the results, project values are given in C$ while oil prices are quoted in terms of US$ for the WTI benchmark. The analysis assumes decision makers act optimally given knowledge about the stochastic process followed by the price of oil, including in which of the two regimes the price resides. In reality the true price process and current regime are not known; however, firms must make decisions based on their beliefs about future oil prices and these beliefs are captured in futures prices which have been used for our price model calibration.[21]

Figure 5 shows the value of the project in each regime for the base case for different starting prices and different resource stock levels prior to any construction expenditures. We observe the project's value rising with price and reserve level in both regimes, as expected. The project has higher values in Regime 1, which again is as expected since this regime has fairly rapid speed of mean reversion to a higher long run mean price than in Regime 2. Note that for lower reserve levels, these diagrams are only for illustrative purposes, as capital costs have not been adjusted to reflect a smaller oil deposit.

Figure 6 shows value versus price, for prices up to US$250. In order to illustrate the optimal decision process, we show the value in each stage *before* the optimal decision is made (before in backwards time). This clearly shows when it is optimal to move from one stage to another. Of course, once we impose the optimal decision, the value function will be

[21] There is a large literature on filtering techniques which are used to uncover unknown parameters, such as regime states, given observed variables, such as futures prices. The Kalman filter is one such widely known approach. Other recursive filtering algorithms have been developed to estimate the parameters of Hidden Markov Models such as in Date et al. (2013). See Mamon and Elliot (2007) for further details. Erlwein et al. (2009) study the use of HMM-based investment strategies for individual portfolio allocation decisions.

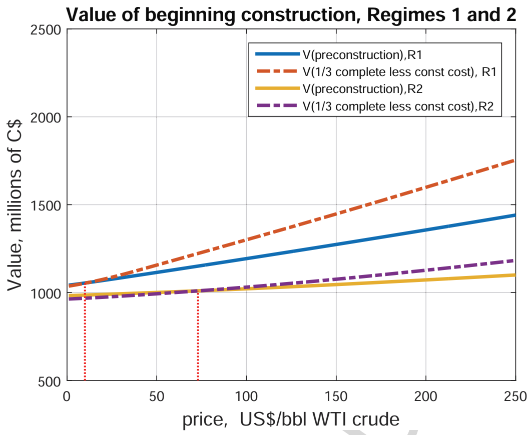


(a) Regime 1 (b) Regime 2

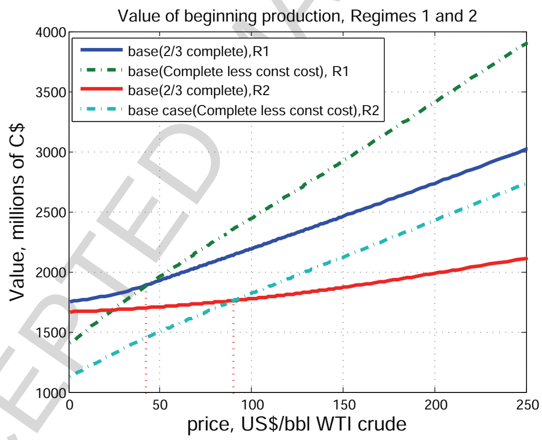
Figure 5: Project value in each regime, C$ millions, versus resource stock size in barrels of bitumen and price in US$/barrel for WTI.

continuous, but non-smooth, since the smooth-pasting condition does not hold for discrete decision times. In Figure 6(a), the value of the project in each regime is shown prior to beginning construction (Stage 1) and once construction has started (Stage 2) less the cost of construction to reach Stage 2. It is optimal to begin construction when the value in Stage 2 less construction costs exceeds the value in Stage 1, which is marked by a dotted red line. In Regime 1, it is optimal to begin construction when the price of WTI is U.S.$10 or higher. In Regime 2, the critical price is much higher at $73. It is optimal to delay construction in the low price regime because of the possibility of switching to the higher price regime and also due to the higher volatility in the low price regime. Figure 6(b) shows a similar diagram for moving from Stage 3 to Stage 4 when production begins. Values are higher since a large portion of capital costs have been incurred. As expected, the critical price for Regime 2 is again higher than for Regime 1.

Table 4 summarizes the critical prices at which it is optimal to move from one stage to the next for the six different stages and for two different reserve amounts. Looking



(a) From Stage 1 to 2



(b) From Stage 3 to 4

Figure 6: Value of beginning construction (Stage 1 to 2) and value of finishing the project and beginning production (Stage 3 to 4). Base case.

first at the columns associated with initial reserves of 250 million barrels, critical prices to move through all phases of the project are higher in Regime 2 than in Regime 1. This implies that the expected time for project completion is longer in the low price regime, and once completed the project is more likely to be in a temporarily moth balled state. Note that once mothballed, the critical prices for reactivation are slightly higher than the prices that caused the decision maker to shut down in the first place. This result implies some

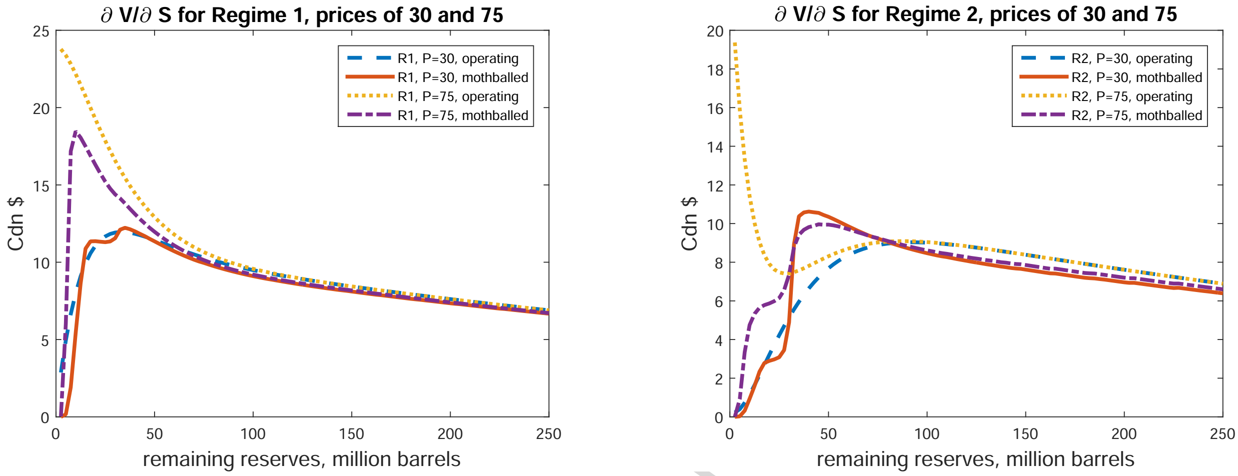
	$S = 250$ mil. bbls		$S = 125$ mil. bbls	
Transition from :	Regime 1	Regime 2	Regime 1	Regime 2
Stages 1 to 2: Begin construction	10	73	45	160
Stages 2 to 3: Continue	25	74	45	117
Stages 3 to 4: Finish, Begin production	42.5	90	56	119
Stages 4 to 5: Mothball	17.5	32.5	25	47.5
Stages 5 to 4: Reactivate	20	37.5	27.5	52
Stages 4 or 5 to 6: Abandon	NA	NA	NA	NA

Table 4: Critical prices for moving between stages, Case 1, U.S. $/barrel, WTI, Full reserve level at $S = 250$ million barrels and half reserves at $S = 125$ million barrels.

persistence in the mothballed state, reflecting the value of the option to delay the irreversible costs of reactivation and production. This phenomenon was highlighted in Dixit (1992) and Mason (2001). The critical prices for mothballing are quite low, which is consistent with observation that operating oil sands facilities have not been moth balled even in the low oil price environment of 2015 and 2016.

It is instructive to compare the columns for the two different reserve levels in Table 4. As noted, we have not reduced capital costs for the lower reserve level, so critical prices for the construction phases are only illustrative. Critical prices at all stages are higher for lower reserve levels. For the construction stages this result reflects the large fixed costs for these types of development. For a producing project this reflects the increasing marginal value of the resource as the stock is depleted - i.e. a larger $\partial V/\partial S$ term in Equation (8). With lower stock levels the operation is more likely to be shut in to preserve the increasingly scarce reserves. Figures 7 plots numerical estimates of $\partial V/\partial S$ versus reserves for the two regimes at two price levels. These diagrams show $\partial V/\partial S$ increases as reserve levels are reduced, reaching a peak at somewhere between 20 to 50 million barrels, depending on the case. For low levels of reserves, in three of the curves shown, $\partial V/\partial S$ declines with reserves due to the large fixed costs of operations which yield economies of scale at low level of reserves.

It is interesting to note that there are no critical prices shown for abandonment for either reserve level shown. Once construction costs have been incurred it is optimal to maintain



(a) Regime 1

(b) Regime 2

Figure 7: Numerical approximation of $\partial V/\partial S$ versus remaining reserves for two prices levels for operating (stage 4) and mothballed (stage 5) projects. Vertical axis is millions of C\$. Horizontal is millions of barrels of reserves.

the project at least in the mothballed state, regardless of price. With the stochastic price process and costs assumed for this analysis, even for very low prices there is still a reasonable possibility that prices will recover and production will again be profitable. However once reserves are significantly depleted it becomes optimal to abandon the project at some positive price level. More details on optimal prices for abandoning the project are given in Section 5.2.3

It may also be observed in Table 4 that for full reserves, critical prices rise as construction proceeds. This is not a particularly intuitive result. The economics of moving from one stage of construction to the next depends on benefits versus the costs of delaying the next capital investment. The benefits of delay include the delay in the spending of construction costs for the next stage. The costs of delay include the delay in receiving revenue from production plus any maintenance costs incurred when construction is paused. The cost of the delay in revenue is stochastic, as it depends on the oil price when the project will be completed. In this case this cost is higher when the project is at an earlier stage of construction, as

	High volatility		GBM with Regime 1 volatility
Transition from :	Regime 1	Regime 2	Single Regime
Stages 1 to 2: Begin construction	5	37.5	124
Stages 2 to 3: Continue	20	42.5	115
Stages 3 to 4: Finish, Begin production	65	143	111
Stages 4 to 5: Mothball	20	54	56
Stages 5 to 4: Reactivate	25	58	58

Table 5: Sensitivity Analysis: Critical prices for moving between stages, U.S. $/barrel, WTI, Full reserve level. High volatility case triples the base case volatilities (σ). Geometric Brownian Motion case is a single regime with a drift of 1% per year and $\sigma = 0.23$.

this implies the decision maker cannot quickly finish construction to take advantage of any potential surge in oil prices. Getting construction underway is like exercising an option that allows the decision maker to move one step closer to a producing project. The significance of this pattern of critical prices is the implication that it may be optimal for producers to begin project development at critical prices below levels that would induce some level of production. As noted, this will be of concern to regulators to the extent that project development itself creates significant environmental damages.

This pattern of critical prices is not a general result, and depends on the nature of the price process involved, and in particular on volatilities and speed of mean reversion. Sensitivity analysis shows that an increase in volatility makes the pattern even more pronounced. Table 5 shows critical prices when the volatility in each regime is tripled. We observe a drop in the critical price to begin construction along with an increase in the critical price to begin production. A high volatility increases the cost of delaying the initial construction phases. The second sensitivity adopts a Geometric Brownian Motion (GBM) price process for a single regime using the Regime 1's volatility. In this case we see a reversal of the pattern with the critical price to begin production at a very high level and then declining for subsequent stages. With a GBM price process the cost of delaying the initial phases of construction is reduced since it is expected that prices will tend up in the long run.

	Case 1 Regime 1	Case 1 Regime 2	Case 2 Weighted Average
η	0.44	1.05	0.73
$\bar{P}$ (US$/bbl)	75	30	53
λ^{jl}	.26	0.28	NA
σ	0.23	0.44	0.33

Table 6: Cases 1 and 2 parameter values. $dP = \eta^j(\bar{P}^j - P)dt + \sigma^j P dz, j = 1, 2.$

	Case 1: Two regimes Base case		Case 2: One regime Wted Average
Transition from :	R1	R2	Single regime
Stages 1 to 2: Begin construction	10	73	0.5
Stages 2 to 3: Continue	25	74	17.5
Stages 3 to 4: Finish, Begin production	42.5	90	40
Stages 4 to 5: Mothball	17.5	32.5	20
Stages 5 to 4: Reactivate	20	37.5	22.5
Stages 4 or 5 to 6: Abandon	na	na	na

Table 7: Critical prices for moving between stages, Comparing cases 1 and 2, U.S. $/barrel, WTI, Full reserve level at $S = 250$ million barrels.

5.2.2. Case 2: The impact of two price regimes

In Case 2 we analyze a single price regime in which price process parameters are specified as a weighted average of the two regimes in Case 1, with weights reflecting the expected length of stay in each regime. The objective is to investigate the extent to which incorporating two price regimes affects optimal decisions. Parameter values for Case 2, with Case 1 provided for reference, are given in Table 6.

Project values before beginning construction (Stage 1) and after the first 1/3 of construction costs have been incurred (Stage 2 less cost) for cases 1 and 2 are shown in Figure 8. Values for Case 2 lie above Case 1 values in either regime. With no risk of switching to a low price regime, critical prices are reduced as can be seen in Table 7. Failure to take account of the two regimes will result in non-optimal actions and incorrect valuation.

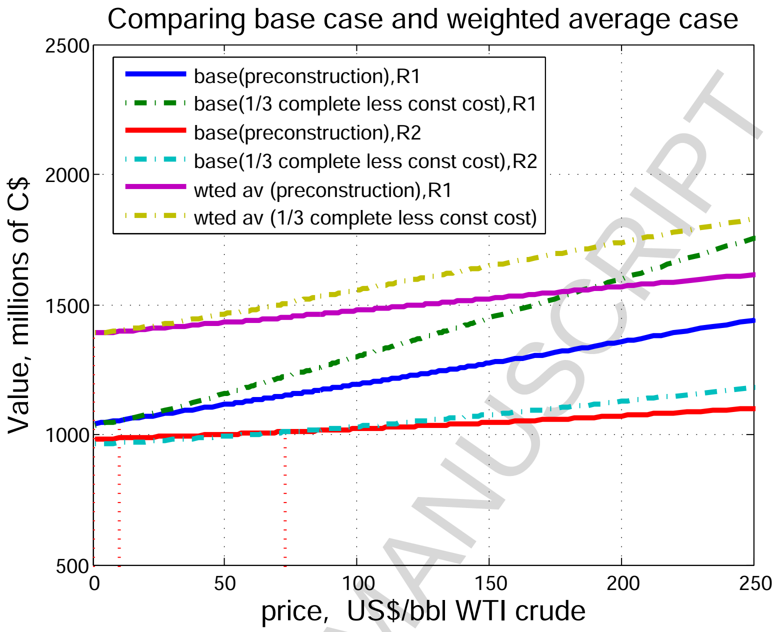


Figure 8: Comparing values of cases 1 (regime switching) and 2 (single weighted average regime) prior to beginning construction and once construction has begun. Solid lines show values before constructions begins. Dashed lines show value after 1/3 of construction costs incurred.

5.2.3. The impact of the carbon tax

Expectations regarding future environmental regulations will have a significant impact on the timing and extent of investment in the oil sands. As a relatively high cost source of petroleum, oil sands investments will be particularly sensitive to any significant strengthening of environmental regulations including policies to restrict carbon emissions, since oil sands production has a high carbon content relative to other sources of oil. McGlade and Ekins (2015) estimate that bitumen production in Canada should be severely curtailed if the world is to maintain a global average temperature at no more than 2 degrees Celsius above pre-industrial times. According to their forecast, if carbon capture storage technology is not available, then bitumen production should cease by 2040.

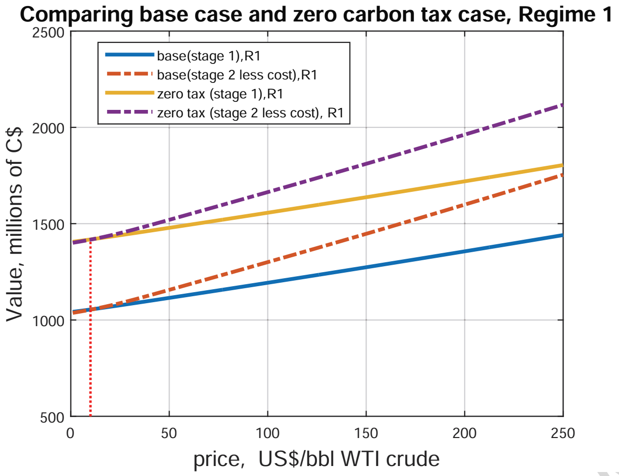
Recall that in the Base Case (Case 1), we adopt the carbon tax scheme announced in Alberta in 2016 - C$30 per tonne carbon tax increasing at 2% per year. This carbon tax was hailed by some, including representatives for some oil sands firms, and soundly criticized by others.[22] Large industrial operations, including oil sands, will be allocated carbon credits and will only have to pay the tax for emissions above their total credits. A total carbon emissions limit of 100 mega tonnes per year from oil sands operations was also imposed, which allows for some growth over current estimated emissions of 70 mega tonnes annually. Even though new and more efficient oil sands operations may not pay any carbon tax under this scheme, the price established for carbon gives an incentive for all producers to reduce emissions to the extent that it can be done at a cost that is less than the tax.

To gauge the impact of this tax scheme, we compare it to a zero carbon tax case (Case 3). A comparison of project values for cases 1 and 3 is provided in Figure 9. There is a significant increase in value but we also find that there is no substantial change in the critical prices at the construction or operations stages. The carbon tax has a significant effect on value because Regime 2 is assumed to have a low long run mean price and is highly volatile, implying the possibility of periods of very low prices over the life of the project which are below the critical price for mothballing the project. In a sensitivity (not shown) the tax has a much smaller effect if the two $\bar{P}$ are assumed to both be above \$50.

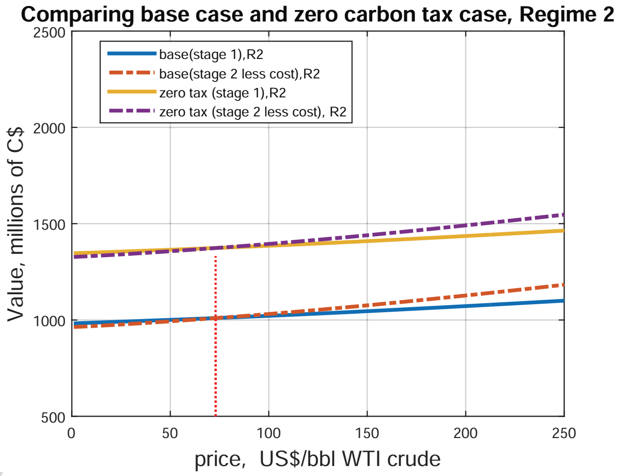
The 2014 report from the Intergovernmental Panel on Climate Change has suggested that a global carbon price increasing to around C\$200 per tonne of CO2 by the middle of this century is needed to mitigate the risk of dangerous climate change (Rivers (2014)). Depending on its implementation, a tax of this magnitude could have a significant effect on both asset value and optimal actions of the oil sands operators. Policy makers face the dilemma of choosing policies consistent with the latest climate science, while not wanting to cause undue harm to an important industry in the economy.

In Figure 10 we show critical prices for the base case plus a range of other different

[22] A rally against the tax in December 2016 drew an estimated 1000 protesters as reported by the CBC news, Dec 3, 2016. CEO's of Cenovus and Suncor argued in favour of the tax.



(a) Regime 1



(b) Regime 2

Figure 9: Project value in each regime for the Base Case ($30 per tonne increasing at 2% per year) and Case 3 (zero carbon tax) , C$ millions, versus oil price in US$/barrel for WTI. Solid lines show value before construction has begun. Dashed lines show value once 1/3 of construction costs are spent.

Case 1 (base case)	Tax starts at $30 per tonne and rises at 2% per year reaching $54.34 by year 30
Case 3	Zero tax
Case 4	Tax increases immediately to $54.34 and remains constant thereafter
Case 5	Tax increases starts at $30 and increases to $200 by year 30
Case 6	Tax increases immediately to $200

Table 8: Details of different carbon tax cases

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carbon tax schemes. A summary of the different carbon tax cases is given in Table 8. In Case 4 increasing the tax immediately to $54 causes critical prices to rise across all stages of operations compared to the base case. The asset value (not shown) declines on the order of 20% but critical prices to begin construction are still quite low in Regime 1. In Regime 2 critical prices increase sufficiently that no new projects are likely to be initiated.

Increasing the tax gradually to $200 per tonne (Case 5) causes critical prices to fall significantly across all operating levels. We observe a type of green paradox whereby moving to this strict regulations causes firms to develop and produce oil sands reserves more quickly than in the base case in order to avoid as much as possible the high tax imposed later in the project. Asset value in Stage 1 drops by about 10% compared to the Base Case.

In Case 6, an immediate increase in the tax to $200 per tonne increases all critical prices substantially. The critical price to begin construction exceeds that of all other stages and is at such a level that no project would be started. Currently operating projects would continue operations in Regime 1 for prices above $40, but would go into mothball phase in Regime 2 for prices below $100.

The different carbon taxes have an effect on the incentive to abandon the project before all reserves have been exhausted. Figure 11 compares critical prices for abandonment for Cases 1, 4 and 6 when the project is operational (Stage 4). Looking first at Figure 11(a) we see that for Case 1 in Regime 1 when the project is operational, if reserves fall to around 40 million barrels or below there is some positive critical price at which it is optimal to abandon the project, meaning that the remaining reserves will never be extracted. For Regime 2, there are positive critical prices from about 65 million barrels and lower. In Case 4 when the carbon tax is increased immediately to $54, critical prices for abandonment are all increased, implying that abandonment before reserve exhaustion is more likely in this scenario. The most stark difference in critical prices happens with the extreme case when the tax is immediately increased to $200 per barrel (Case 6). In Regime 1 for a wide range of reserves, an operating project would be abandoned if the WTI price is around $40 or less. Over a similar range of reserves, in Regime 2, the project would be abandoned for a critical price of $100.[23] The sudden imposition of this high tax will likely result in a large portion of reserves going unexploited. Note that critical prices for abandonment for Case 5 (gradual increase to a $200 tax over 30 years) are not shown as these are very close to those of the Base Case.

Of course, all of these critical prices are only illustrative as they depend on the many assumptions made in the analysis including projects specifications and the assumed price process. However the analysis does illustrate the sensitivity of the oil sands project to different carbon tax schemes and the starkly different optimal strategies in the presence of two price regimes. From a policy perspective, the analysis illustrates that the gradually rising carbon tax has some negative environmental impacts as firms are motivated to speed up development of reserves, thereby increasing atmospheric carbon concentrations. In contrast a sudden tax increase would motivate firms to delay project investment and the expected total quantity extracted would be reduced. The incentives provided with a gradually increasing tax are consistent with the concerns raised by the so-called Green Paradox. The practical significance of the Green Paradox in a general equilibrium setting is a topic of is still under debate. Nevertheless, this modelling exercise suggests that at the firm level, the incentive to speed up development to avoid future tightening of environmental regulations has an significant effect. Although the example shown here is for carbon regulations, the same intuition would apply to other sorts of regulations, such as gradually increasing requirements for monitoring, abatement of emissions and remediation of environmental damages.

[23]The graphs have been cut off at 140 million reserves to make them easier to read. In Case 6 the critical prices for abandonment at reserves of 250 million barrels are $32 for Regime 1 and $96 for Regime 2.

6. Concluding Remarks

We argue that a regime switching price process is a logical choice for capturing the boom and bust cycles of commodity prices and the associated non-constant drift and volatility parameters. We have calibrated a regime switching price process for crude oil with two mean reverting price regimes and considered the implications for optimal development of a prototype resource extraction project. The calibration, based on oil futures prices over the last twenty years, shows two mean reverting price regimes, one with a significantly higher long run average price than the other. Optimal actions as determined by critical prices were found to differ significantly between the two regimes. The calibration process is sensitive to various factors including the time frame for the data and upper and lower limits specified for parameter values. However the analysis provides useful insight into optimal resource extraction decisions in the context of highly uncertain commodity prices, which should help inform regulators charged with mitigating the environmental consequences of such projects.

A focus on the paper has been on the pace of construction as implied by the pattern of critical prices This is important to the extent that there are irreversible environmental damages happening during the construction phase. Regulators should be aware of the potential ramp up in resource development activity in response to movements commodity prices and the need to be prepared in terms of having an adequate regulatory framework in place. A criticism of resource management practices in the recent past is that environmental regulations lag the pace of resource development.

For our base case we observed a low initial price to begin construction, with critical prices rising through the construction phases. This implies it is optimal for firms to begin project development even if prices are not yet at a level which would make production economic. Paying for the initial stage of construction buys the option on a project that can be started in two years time. Paying for the next step buys the option on a project that can be started in one year's time. This pattern of rising critical prices over the phases of construction was found to depend, in particular, on the speed of mean reversion and volatility. The pattern was exacerbated for processes that were more strongly mean reverting and more volatile as these increased the cost of delaying the early construction phases.

We also investigated the effect of various carbon tax schemes on optimal decisions. We compared a carbon tax based on the 2016 Alberta proposal ($30 per tonne rising at 2% per year) with a tax that increases to $200 per tonne by year 30. Critical prices for the latter case were lower than for the former, implying that the reserves would be developed and produced more quickly in the latter. In contrast, a sudden increase of the tax to C$200 per tonne raised critical prices at all of the construction stages to such an extent that no new oil sands investment would occur. In addition, currently operating projects were more likely to be abandoned before reserves had been exhausted.

From an environmental perspective, the incentive for firms to speed up development and production is clearly a disadvantage of the gradualist approach to increasing the cost of carbon. This is a demonstration of one avenue through with the Green Paradox may operate. However, a gradual increase of a carbon tax is much more favourable to resource producers, as it gives them time to adjust their actions in response to the changes in the tax. On a more macro scale, this also makes the required economic transition less painful for the regional economy. The knowledge that carbon taxes will rise over time will spur innovation to make resource extraction less carbon intensive, which has been a major focus of the oil industry over the past decade. Modelling innovation in extraction techniques is outside of the scope of this paper, but would represent an important future extension of this research.

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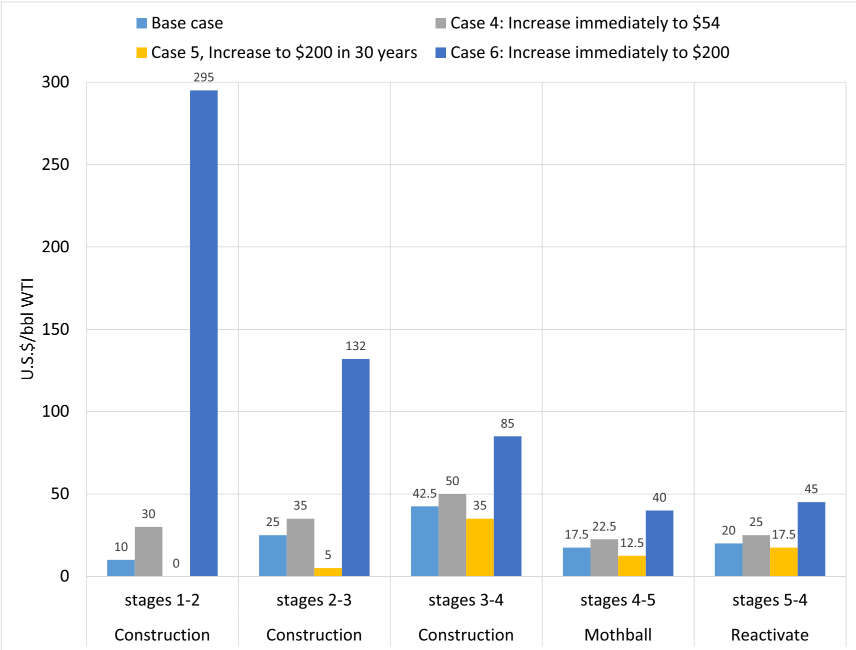
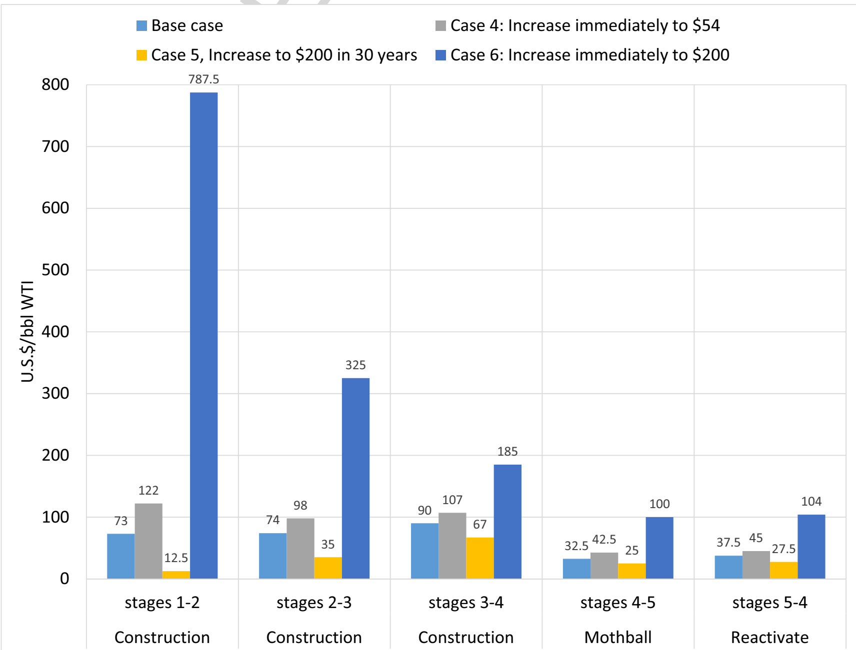
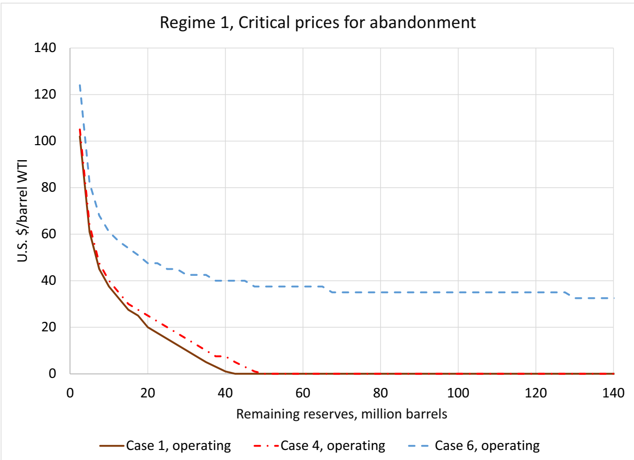


(a) Regime 1

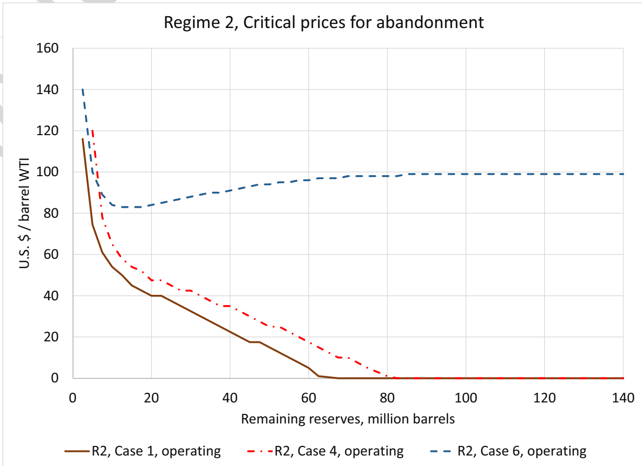


(b) Regime 2

Figure 10: Critical prices for various carbon taxes schemes



(a) Regime 1



(b) Regime 2

Figure 11: Prices for project abandonment versus remaining reserves. Case 1 (base case which includes a carbon tax of $40 per tonne), Case 4 (carbon tax of $54 per tonne), and Case 6 (carbon tax of $200 per tonne).

Highlights

- Oil prices are modelled as a stochastic regime switching process to capture the market uncertainty faced by firms making investment and production decisions regarding oil sands extraction.
- A typical firms optimal decisions are examined as a stochastic optimal control problem and a numerical solution is implemented.
- Regime shifts, between boom and bust prices, have an important effect on the timing of optimal extraction and production.
- A gradually increasing carbon tax accelerates the firms optimal pace of oil extraction, while a sudden increase reduces the pace of extraction and causes a project to be abandoned sooner.